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# Optimal flow rate control in solar thermal system for domestic hot water preparation

**Domenico Marano[1], *Vinod Kumar Sharma[1], Joginder Singh[2]**

[1]ENEA, Research Centre, Trisaia, 75026, Rotondella (MT) Italy
[2]Department of Horticulture, J. V. College, Baraut, UP, India
***Corresponding email**: *vinodkumar.sharma@enea.it*

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**Article Authors**
Domenico Marano,
Vinod Kumar Sharma, Joginder Singh

**Corresponding Author Email**
*vinodkumar.sharma@enea.it*

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**ABSTRACT**

The paper deals with optimal control of flow rate in a solar thermal system used for the production of hot water. The main objective is to maximize the net savings from the plant obtained in terms of useful energy collected and cost of energy spent for pumping of the thermal fluid. It is in the above context that based upon the annual simulation performed using the controller introduced herewith (Byron Winn and Hull, 1979), an estimation of annual economic savings that an optimally designed flow rate controller can provide when compared to classic on - off control in climatic conditions of the Mediterranean countries, has been discussed in the present paper.

**KEYWORDS**

Optimal Flow Rate Control, Solar Thermal System, Domestic Hot Water

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Currently, in most cases, control of solar thermal systems is on-off type with fixed flow rate. The strategy is simple and ensures that the collector produces useful energy when the weather conditions and temperatures difference between boiler and collector allows it. Moreover, this control strategy allows avoiding the dissipation of energy into the environment when the boiler temperature is higher than the output from the collector. In our case, determination of flow rate, instead, is obtained applying theory of optimal control based on the principle of the maximum, set out in 1960 by the Russian mathematician Lev Pontryagin (Locatelli, 1996). An overview of the literature relevant to the applications of optimal control theory in solar thermal systems is given in (Badescu, 2008).

It is worth to note that the contributions of (Kovarik and Lesse, 1976, Byron Winn., Hull, 1979, Saltiel and Sokolov, 1985 and Badescu, 2008) are particularly important, for optimal control problem, discussed here. The control variable is the fluid flow rate through the collector. The objective functional to be minimized is the index J, given by:

$$\boldsymbol{J} = \int_{t_0}^{t_f} (\boldsymbol{P_p} - \boldsymbol{Q_u})\,\boldsymbol{dt}$$

Where, $\boldsymbol{P_p}$ is the mechanical power of the pump, $\boldsymbol{Q_u}$ is the useful thermal power supplied by the solar system and $[\boldsymbol{t_0}\ \boldsymbol{t_f}]$ is the time interval between sunrise and sunset.

In Kovarik's work, the system of differential equations based on the principle of the maximum is resolved numerically by obtaining the value of the optimal control according to time. In Winn's work with a series of simplifications, the expression of the optimal flow at a given instant is obtained according to measurable parameters such as radiant and ambient temperature relative to that instant, and the values thus obtained are compared with those of the Kovarik, showing an excellent agreement between the two methodologies. In Saltiel's work, a solar system is being studied with collectors of different types connected in series. It is well known fact that a string of collectors with non-selective absorber plate when compared to collectors with absorber plate with selective coatings, allows to increase the thermal yield. Needless to say those non-selective collectors at low temperature have a higher yield, while at higher temperatures, the opposite occurs.

Even in the present publication, the principles based on the principle of the maximum, are solved numerically and different load profiles are considered to evaluate the influence that daily load distribution has on determining the optimal flow rate. In other words the aspect that makes this work different from the publications mentioned above is the choice of a different objective functional to be maximized, *i.e.* the net energy saving of the system defined as the difference between the saving due to the useful energy delivered by the system and the cost of energy for pumping the heat transfer fluid. The result is that the load profile has little influence on both the optimum flow rate and minimum value of the index “J”.

Finally, in the work of Badescu, collector model with more details is used compared to the previous ones, different load profiles are considered and, also in this case, numerical methods are used for the solution of the system of differential equations based on the principle of the maximum. In view of this, a calculation algorithm, based upon the maximum principle, has been developed to identify the flow rate pattern that maximizes the net annual savings achievable from a solar system under the climatic conditions of southern Italy. This in order to assess quantitatively the benefits of the optimal control strategy compared with the on-off control.

The most salient feature of work is that instead of carrying out the analysis for a few days, as in the work reported in (Kovarik, and Lesse, 1976, Byron Winn. and Hull, 1979, Saltiel and Sokolov, 1985) simulation extended over the whole year has been performed using measured experimental data of radiance and ambient temperature. In order to achieve a sufficient accuracy in calculations, different numerical solution methods and various integration steps have been implemented. The analysis has been carried out using the hourly weather data of ENEA Solar Energy Laboratory test site and solving the differential equations describing the system. Finally, in order to evaluate the increase in annual energy saving, achievable with the optimal flow rate control compared to the on-off control, an appropriate performance index has been introduced and evaluated for different values of the specific flow rate. The innovative points in this paper compared to the above mentioned publications have been summarized as follows:

- The index to be maximized consists of the net savings of the system, comprising of the cost of methane saved less the cost of electricity spent to circulate the heat transfer fluid instead of the net energy savings.
- The analysis has been conducted on annual basis using the weather data available at Trisaia site and solving the differential equations using the Second Runge Kutta method (Pasquini, 1976) with integration step of 112 sec (Appendix A2). Numerous methods of solution (Runge Kutta method of second and fourth order) were then compared from the point of view of calculations accuracy as function of the integration step.
- Optimum flow rate calculations have been done by iteratively solving the equation (Pasquini, 1976) without introducing approximations in the expression of the thermal removal factor and its derivative
- An index was introduced to evaluate the annual savings gains with optimum flow control versus on-off control for various values of the specific flow rate of the latter.
- Based upon the result obtained from the simulations, it was possible to conclude that there is scope for “on-off” control that equals the savings obtainable with the optimal strategy.

The discussion will be focused referenced to system with direct connection between boiler and collector (Kovarik and Lesse, 1976, Byron Winn., and Hull, 1979, Saltiel and Sokolov, 1985). The appendix will be used to show the corrections to be made to the algorithm, in case of system with immersed coil.

**Development of Mathematical Model for Solar System Investigated**

The plant under investigation is schematically represented in figure 1. This is a system where the connection between boiler and collector is direct. The presence of the immersed coil will be discussed later in appendix showing that the control algorithm developed in the absence of the exchanger remains valid provided few modifications are made. For the operation of the system, it is assumed that the user will require a certain amount of water at a temperature around 45 °C. In case of the boiler temperature above 45°C, water from the boiler will be mixed with the mains water (through the mixing valve) to lower its temperature to the set point value. In this case, the boiler will remains off. If the boiler water temperature is below the set point of 45 °C, the mixing valve does not intervene and water supplied to the user will be heated by the boiler up to 45 °C. Mathematical model of the solar system, is illustrated, below, *i.e*.

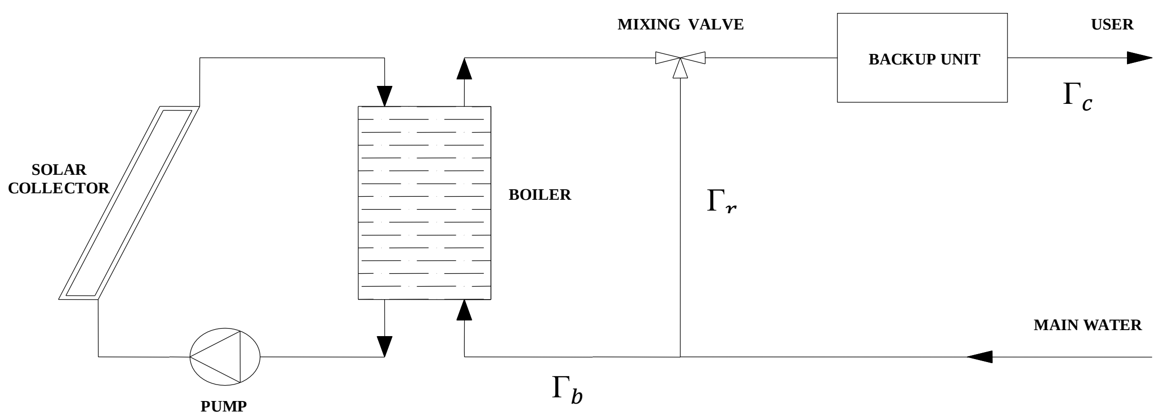


**Fig 1. Schematic diagram of the solar system**

Considering that the water in boiler is completely mixed at temperature Ts and there are no heat losses from the boiler, it is possible to define the energy balance as given below:

$$\frac{dT_s}{dt} = \frac{Q_u}{C_s} - \frac{Q_{load}}{C_s} \qquad \mathbf{1})$$

Where, $Q_{load}$ (W) is the thermal load and $C_s$ (J/°K) is the heat capacity of the boiler.

The power output $Q_u$ (W) is given by the equation (Duffie and Beckman, 2006):

$$Q_u = F_r\, A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)] \qquad \mathbf{2})$$

Where.

$\boldsymbol{A}$: Aperture area (m²)

$\boldsymbol{K_{\tau\alpha}}$**:** Incidence angle modifier

$(\boldsymbol{\tau\alpha})$**:** Transmittance - absorptance product at normal incidence

$\boldsymbol{G}$**:** Global solar irradiance on the collector plane (W/m²)

$\boldsymbol{U_c}$**:** Overall heat loss coefficient (collector + boiler) (W/m²K)

$\boldsymbol{T_a}$**:** Surrounding air temperature (°C)

The heat removal factor $F_r$, is an increasing function of flow rate and its analytical expression is given below (Duffie and Beckman, 2006):

$$F_r = \frac{\dot{m}\, c_p}{A\, U_c}\left[1 - exp\left(-\frac{A\, U_c\, F'}{\dot{m}\, c_p}\right)\right] \qquad \mathbf{3})$$

Where,

$\dot{m}$ **:** Flow rate of the vector fluid (kg/s)

$\boldsymbol{c_p}$: Specific heat of the vector fluid (J/kg K)

**F':** Collector efficiency factor (expression can be found in (Duffie and Beckman, 2006).

As shown in fig 1, by imposing the conservation of mass and energy for the mixing valve, it is possible to obtain:

$$\Gamma_b = \Gamma_c \frac{T_{set} - T_r}{T_s - T_r} \quad per\ T_s > T_{set}$$
$$\Gamma_b = \Gamma_c \qquad per\ T_s < T_{set}$$

Where,
$T_r$: Main temperature (°C)
$T_{set}$: Set water temperature required by the user (°C)
$T_s$: Temperature of boiler (°C)

Finally, the thermal load required by the user is given by:

$$Q_{load} = \Gamma_b c_p (T_s - T_r) \qquad 4)$$

**Application of the Maximum Principle to the Case Study**

In the present case study, the index to be minimised is given below:

$$J = \int_{t_0}^{t_f} (c_e P_e - c_g Q_u) dt \qquad 5)$$

Where, [$t_0$ $t_f$] is the time between dawn and sunset, $c_e$ is the cost of electricity (Euro/J) and $c_g$ the cost of fuel used in the backup unit, *e.g.* methane (Euro/J) and control is done through flow rate in the collectors, as shown in (Kovarik and Lesse, 1976, Byron Winn. and Hull, 1979, Saltiel and Sokolov 1985, Badescu, 2008). Minimization of functional "*J" (*equivalent to maximization of the net savings of the system) defined as the difference between the useful energy delivered by the solar system and the cost of the energy spent for pumping the fluid vector, is the most obvious interpretation of equation (5). The instantaneous power output is given by equation (2), while the electrical output of the pump is given by (Kovarik and Lesse, 1976, Byron Winn. and Hull, 1979, Saltiel and Sokolov, 1985, Badescu, 2008):

$$P_e = k\dot{m}^3 \qquad 6)$$

Where, *k* is a characteristic parameter of the pump, which depends on the pressure drops of the collector's loop, whose calculation is shown in (Badescu, 2008).

The search for the optimal flow rate is made by applying the maximum principle. In view of this, the function Hamiltonian, *H* is given by:

$$H = Q_u \left( \frac{p}{C_s} - c_g \right) - \frac{p}{C_s} Q_{load} + c_e P_e \qquad 7)$$

The application of the maximum principle illustrated referred to (Locatelli, 1996), leads to the following system:

$$\frac{dT_s}{dt} = \frac{\partial H^T}{\partial p} \qquad 8a)$$
$$\frac{dp}{dt} = -\frac{\partial H^T}{\partial T_s} \qquad 8b)$$
$$\frac{\partial H}{\partial u} = 0 \qquad 8c)$$
$$T_s(0) = T_{s0} \qquad 8e)$$
$$p(t_f) = 0 \qquad 8f)$$

Where, *p* is the auxiliary variable, commonly called co-state and T is for transposition.

In order to apply such formulas to the present case, it is necessary first to calculate the derivative of H with respect to the control input u (in our case the optimal flow m ̇_0p). Taking into account that $Q_{load}$ does not depend on m ̇_0p, we get

$$\frac{\partial H}{\partial \dot{m}_{op}} = \left( c_g - \frac{p}{C_s} \right) \frac{\partial Q_u}{\partial \dot{m}_{0p}} + c_e \frac{\partial P_e}{\partial \dot{m}_{0p}} = 0$$

$$\frac{\partial Q_u}{\partial \dot{m}_{0p}} = A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)] \frac{\partial F_r}{\partial \dot{m}_{0p}}$$

$$\frac{\partial P_e}{\partial \dot{m}_{0p}} = 3k\dot{m}_{op}^2$$

Substituting these expressions in 8c) we get 9c whereas replacement of $Q_u$ and $Q_{load}$ in (1) by expression as per (2) and (4) respectively results in (9a). Finally, substituting in (8b) the expression (7) of H and derivation w.r.t. $T_s$, it is possible to obtain (9b).

Ultimately, for the problem under investigation, below given system will applies:

$$\frac{dT_s}{dt} = \beta 1 \frac{F_r(\dot{m}_{0p})A}{C_s}[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)] - \frac{\Gamma_b c_p(T_s - T_r)}{C_s} \qquad 9a)$$

$$\frac{dp}{dt} = \frac{F_r(\dot{m}_{0p})AU_c}{C_s}p - F_r(\dot{m}_{0p})AU_c c_g \qquad 9b)$$

$$\dot{m}_{0p} = \sqrt{\beta 1 \left(\frac{1}{3kc_e}\left(c_g - \frac{p}{C_s}\right)A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)]\frac{\partial F_r}{\partial \dot{m}_{0p}}\right)} \qquad 9c)$$

$$T_s(0) = T_{s0}$$
$$p(t_f) = 0$$

Where,
$\dot{m}_{0p}$**:** is the optimal flow rate (kg/s)
$T_{s0}$**:** is the temperature of the boiler at the sunrise (°C)
$\beta 1$ : is a coefficient that is equal to 1 when $(K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a))>0$ and it is equal to 0 when $(K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a))<0$. In order to solve the system of equations, it is necessary to express the term $\frac{\partial F_r}{\partial \dot{m}_{0p}}$ as function of $\dot{m}_{0p}$.

To this end, applying the rules for derivation of composite functions and product functions to (3) is obtained, it is possible to obtain;

$$\frac{\partial F_r}{\partial \dot{m}_{0p}} = \frac{1}{\dot{m}_{0p}}\left(F_r(\dot{m}_{0p}) - \mathrm{F'e}^{-(\mathrm{F'}\frac{A\,U_c}{\dot{m}_{0p}\,C_p})}\right) \qquad 9d)$$

With reference to the above-cited formulas, it is possible to note the following. There are three equations in three unknowns (system 9 in which $\frac{\partial F_r}{\partial \dot{m}_{0p}}$ is expressed by 9d): $T_s$, $p, \dot{m}_{0p}$ accompanied by the contour conditions relative to the initial temperature and the value of the co- final status.

Its solution can be obtained numerically (Kovarik and Lesse, 1976, Saltiel and Sokolov, 1985, Badescu, 2008) with attainment of optimal control over time but with prior knowledge of G, $T_a$ and $Q_{load}$ over the entire time interval [$t_0$ $t_f$]. In other words, to determine the optimal flow at the instant "t", it is necessary to know the future values at instant "t" of radiant t, ambient temperature and load which obviously are not available. It can therefore be noted that mainly because of these reasons, practically, it is not possible to realize a controller using this approach. Its usefulness lies in the fact that it is possible to determine the maximum savings obtainable comparable to the ones achievable with different strategies thus helping to decide the ones closer to the optimum situation. Following the discussion reported in (Byron Winn and Hull, 1979), it is to be noted that if for the usual values of parameters in equations 9), results

$$c_g \gg \frac{p}{C_s}$$

Then by substitution of (9d) in (9c) and taking into account (3), the optimum flow is given by:

$$\dot{m}_{0p} = \sqrt{\beta 1\left(\frac{c_g}{3kc_e}A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)]\frac{1}{\dot{m}_{0p}}\left(\frac{\dot{m}_{0p}c_p}{A\,U_c}\left[1 - \mathrm{e}^{-(\mathrm{F'}\frac{A\,U_c}{\dot{m}_{0p}\,C_p})}\right] - \mathrm{F'e}^{-(\mathrm{F'}\frac{A\,U_c}{\dot{m}_{0p}\,C_p})}\right)\right)} \qquad \mathbf{(10)}$$

This equation is of the type:

$$x = f(x) \qquad 10a)$$

Solution to (10a) is a number $\xi$ that results to be

$$\xi = f(\xi)$$

This solution is obtainable by iterative methods. The one that will be used here, referred to as the combined point or subsequent replacements (Pasquini, 1976) is given below:

$$x_{i+1} = f(x_i) \qquad 10b)$$
$$i = 0, 1, 2 \ldots\ldots\ldots$$

It can be shown (Pasquini, 1976) that if the succession obtained by applying the combined point algorithm is convergent, this is sufficient to assert that the limit to which succession (10b) converges is the solution of (10a). In our case, the convergence condition of algorithm (10b) applied to (10) was satisfied and therefore the obtained iterative solution actually provides a solution of (10). Moreover, the second term of (10) being a continuous, monotonous and decreasing function of $\dot{m}_{0p}$, it can be considered to be a unique solution. All this is clear from figure 2 where f (x) is the second member of (10), x is the flow rate and of course the solution is given by the intersection of the two curves. From the calculus accuracy point of view, the interactions stop when the desired approximation is reached using conditions of the type (Pasquini, 1976):

$$|x_{i+1} - x_i| < \delta$$

With δ sufficiently small the controller thus introduced is practically feasible as expressed in terms of measurable parameters such as instantaneous values at time t of $G$, $T_a$ e $T_s$. In (Dorato and Jamshidi, 1982) it has been proven that inequality.

$$c_g \gg \frac{p}{C_s}$$

With usual values of the parameters in many cases not valid. On the other hand, it was found that results obtained by applying (10) lead to a value of index “J” practically coincident with the one obtainable by resolving, without introducing approximations, the system (9). This is because of the fact that the mistakes in the first part of the day are nearly the same in form but opposite in sign compared to those in the second part, so they end up compensating. It can therefore be concluded that control (10) allows achieving the same savings obtainable with the optimal control given by the system solution (9).

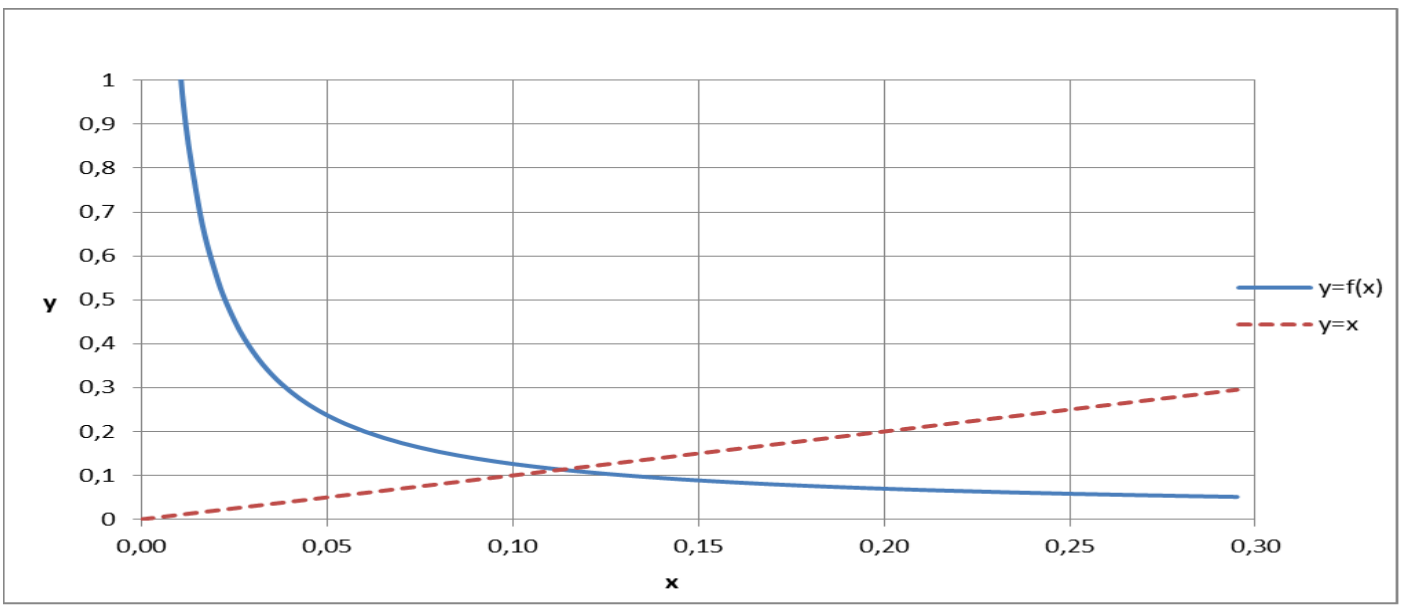


**Fig 2. Graphical solution of equation 10**

**Simulation Results**

The simulations on the system of figure 1 have been performed under the same assumptions considered in (Byron Winn and Hull, 1979). The main parameters used in the present investigation are summarised in the (table 1) given below:

**Table 1. Main parameters for investigation**

| $(\tau\alpha)$ | ***U<sub></sub>***$U_c$ [W/m²K] | *k* [W/(kg/s)³] | $A_a$ [m²] | # of Collectors | $C_s$ [MJ/K] |
|---|---|---|---|---|---|
| 0.70 | 4 | 1000 | 2 | 28 | 19 |

Where,
**K:** pump coefficient
**Aa:** collector aperture area
**Cs:** thermal capacity of storage

The simulation has been carried out on annual basis, considering hourly values of *G* and $T_a$. Moreover, a load profile characterized by a single water withdrawal during the evening equal to 50 litres per user with a total number of users equal to the surface of the solar field (56 users), has been chosen. A series of simulations were made on an annual basis using the time values of G and Ta by solving the system 9) with the following algorithm: At time, t is resolved with the combined method of as shown in (10) while introducing the values of G, Ta and Ts. Integrating equation 9 a) using the values of m ̇_0p calculated at the previous point, so as to obtain the Ts and Q_u for the next instant t + 1. In the simulations two control strategies have been compared: the first characterized by a traditional on-off control with values of flow rate ranging from 0.001 to 0.02 kg/s m² and the second with the optimal flow rate control. For comparison purpose, net savings parameter achieved on annual basis, named *R*, has been considered. Its expression is given by the following formula, where the integration is extended to the entire year:

$$\boldsymbol{R = \int c_g Q_{load} dt - \int c_e P_e \, dt} \qquad \textbf{(11)}$$

Calling $R_{opt}$ the annual savings achievable with the optimal control strategy and $R_c$ those achievable with the on-off control strategy, the percentage index quantifying the advantage of the optimal control, with respect to the on-off, is given by:

$$I = \mathbf{100}(\boldsymbol{Ropt - Rc})/\boldsymbol{Rc} \qquad \textbf{(12)}$$

The graph of figure 3 shows the trend of this index by varying the values of specific flow rate in the on-off functioning. The graph shows that the maximum possible saving with the on-off control is achieved for a specific flow rate of about 0.007 kg/s m² and is equal to that achieved with the optimal strategy. In general, the flow rates used in conventional solar systems are higher than the value obtained previously and usually are comprised between 0.01 and 0.02 kg/s m². In this interval the index I rise significantly: the net energy savings with optimal control strategy is, in fact, 1.5% higher if compared to those achievable in on-off control with a specific flow rate of 0.01 kg/s m². While, with a specific flow rate in the on-off control equal to 0.02 kg/s m², the increase of savings that can be achieved with optimal control, rises to 31%. These data shows that the functioning with optimal control allows to attain considerable savings compared to the on-off control in case of high values of the specific flow rate for the latter. In absolute terms, a saving of about 3415 Euros/year for the optimal flow rate control has been found. With a specific flow rate of 0.014 kg/s m² in the on-off control (typical of conventional solar systems), the savings with optimal strategy is about 235 Euros/year if compared to the on-off control.

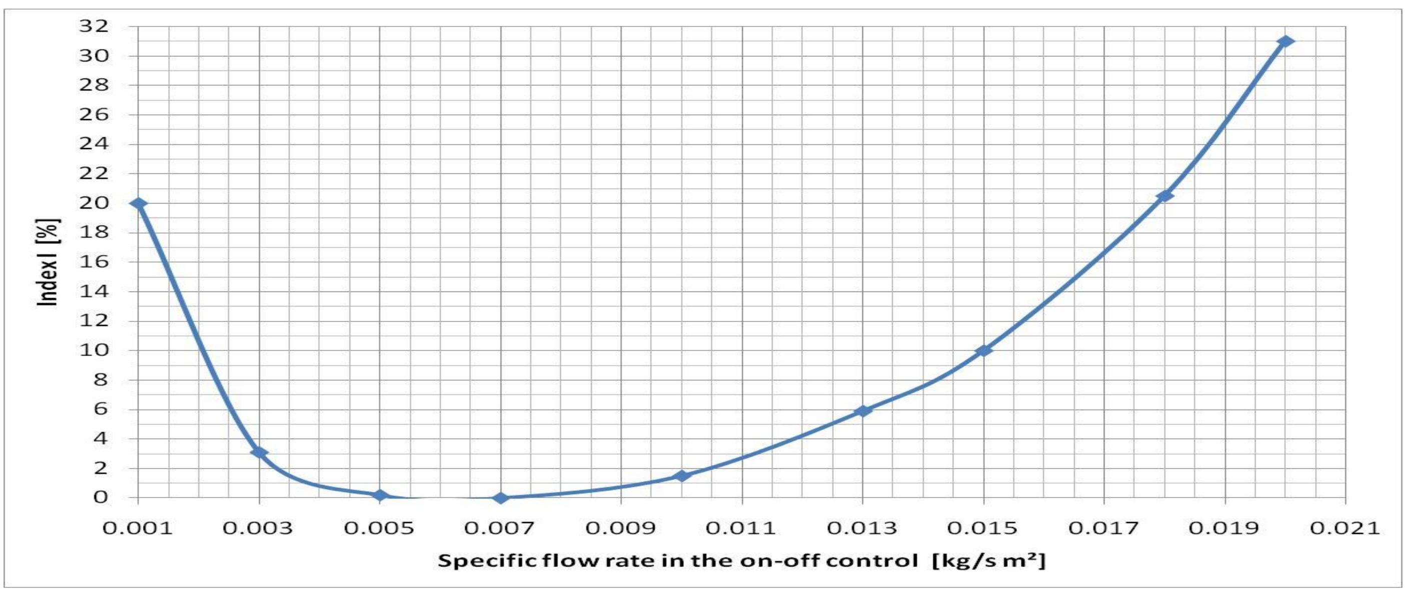


**Fig 3. Trend of index I by varying the specific flow rate in the on-off control strategy**

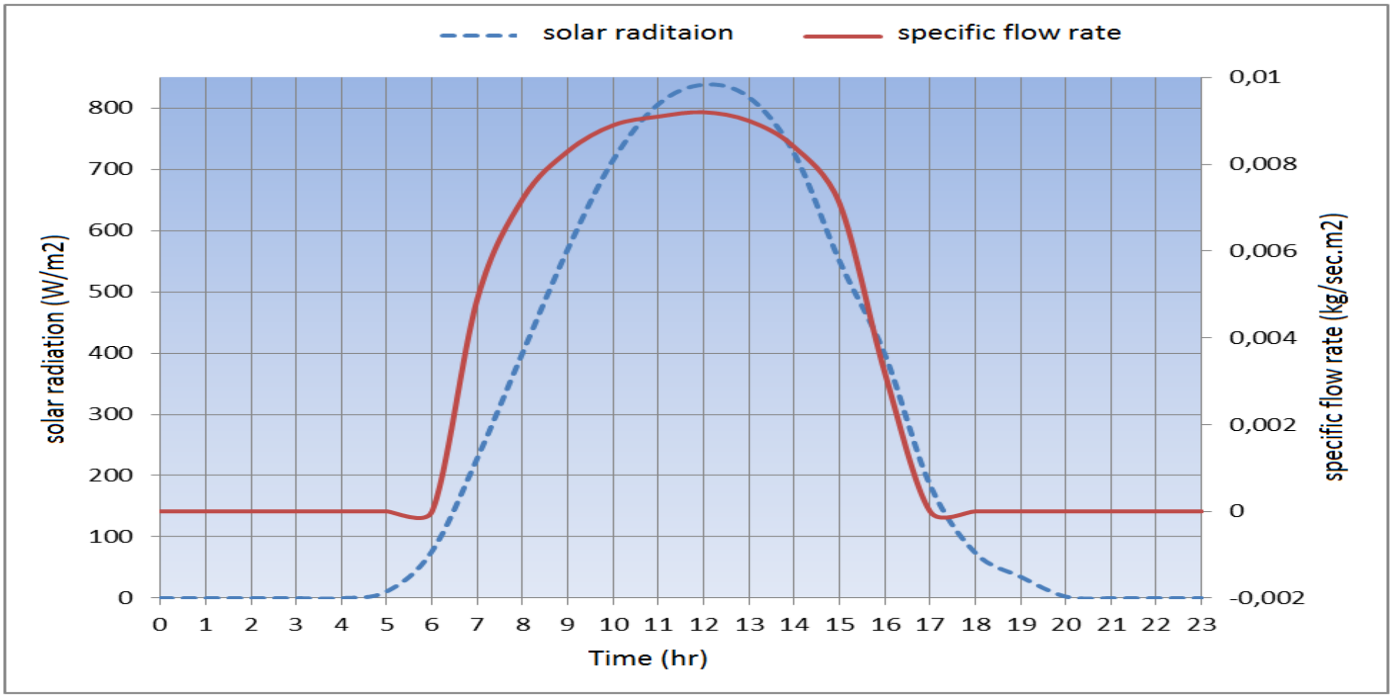


**Fig 4. Solar radiation and specific optimal flow rate for a typical summer day**

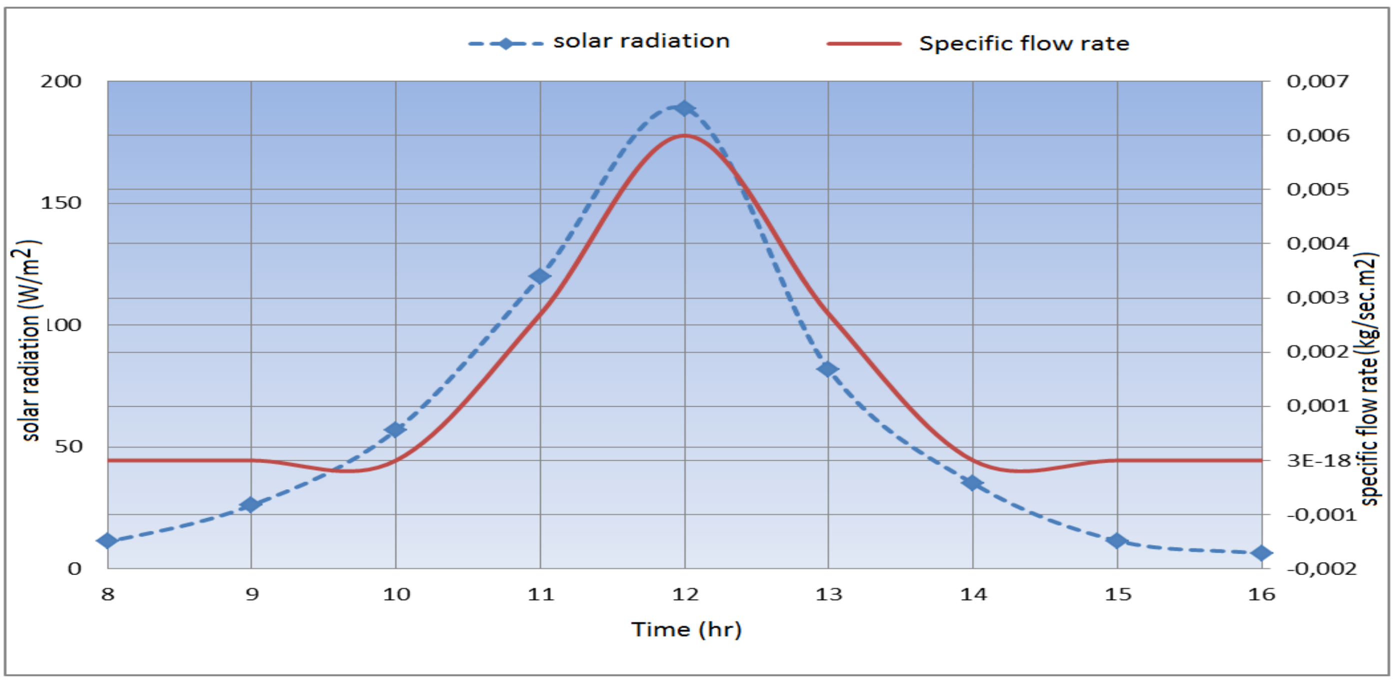


**Fig 5. Solar radiation and specific optimal flow rate for a typical winter day**

Figure 4 shows a typical trend of the optimum flow rate and relative trend of radiance for a summer day. It shows that the optimum flow follows the trend of the radiance and remains below the typical values used in on - off operation (the latter typically ranging from 0.012 to 0.020 kg / sec $m^2$). Note that the pump will be “ON” when the radiance exceeds 150 W / $m^2$ and it shuts off when the radiance drops below this value. The figure shows a slight dissymmetry of the flow curve which is explained by the fact that in the morning the boiler is cold and therefore able to collect useful energy with low radiance values. The boiler then warms up during the day and therefore increases the collector losses, so that higher power is needed to collect useful energy.

Figure 5 shows the case of a covered winter day. It is obvious from the figure that the pump is also running at low radiance values with lower flow rates than the summer case and that the ignition threshold is about 50 W / m$^2$.

## Conclusion

A calculation algorithm based on the principle of maximum has been introduced for the identification of the flow rate pattern that maximizes the annual energy savings attainable from a solar thermal system located at the ENEA Trisaia centre, quantitative evaluation of the benefits of optimum strategy compared to "on – off" control. The algorithm is able to assess quantitatively the benefits of optimal strategy with respect to the ordinary on-off strategy for the control of the flow rate in the collectors loop. It is evident from the results obtained that the use of optimal control strategy, based on the maximum principle, results in a significant increase of the energy savings compared to on-off control. The calculations show that the increase in savings achieved with the optimal strategy varies from 0% (for a specific flow in on - off operation of 0.007 kg/sec m$^2$) to 31% (for a specific flow in the on - off operation of 0.02 Kg/sec m$^2$). From this analysis, it also emerges that the on-off exercise with streams lower than the traditional ones (typically 0.005-0.007 kg/sec m$^2$ vs. 0.01-0.02 kg/sec m$^2$) allows to have performances very close to those obtainable with the strategy Optimal.

In addition, the simulations carried out have revealed that, with the on-off control, flow rates lower than those traditionally used in solar heating systems, allow attaining energy performance very close to those achievable with the optimal control strategy. The analysis carried out showed that it is possible to implement the feedback of an optimal controller through the measurement of radiance, room temperature, boiler temperature, flow rate, using a pump controlled by variable frequency inverters. Naturally, all of this has a cost that when compared with the greatest savings gained with the classic on - off control, helps to decide whether or not the control is good or not. From this point of view, taking into account 9c), it is recognized that as an alternative to the very expensive radiation measurements, it is possible to simply measure the output temperature of the collector Tu (Byron Winn and Hull, 1979) as it is of equal value:

$$A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)] = \dot{m}_{0p}C_p(T_u - T_s)/F_r$$

## Appendix A1

If there is a coil exchanger immersed with efficiency ε in the system, changes to the algorithm introduced in this publication must be made. First of all, in the expression of Qu, we need to introduce the modified 'removal factor $F_r'$, to take account of the exchanger instead of $F_r$ (Duffie and Beckman, 2006):

$$Q_u = F_r' A[K_{\tau\alpha}(\tau\alpha)G - U_c(T_s - T_a)] \qquad \textbf{(A1)}$$

Where, modified removal factor $F_r'$ is given by (Duffie and Beckman, 2006)

$$F_r' = \frac{F_r}{1 + \frac{F_r A U_c}{m C_p}\left(\frac{1}{\varepsilon} - 1\right)} \qquad A2)$$

In addition, by applying the usual rules for derivation of composite functions and function product, 9d) is replaced by:

$$\frac{\partial F_r'}{\partial \dot{m}_{0p}} = \left(\frac{F_r'}{F_r}\right)^2 \frac{\partial F_r}{\partial \dot{m}_{0p}} + F_r'^2 \frac{A U_c}{\dot{m}_{op}^2 C_p}\left(\frac{1}{\varepsilon} - 1\right) \qquad (A3)$$

It follows that in order to take account of the exchanger, just replace the algorithm, valid for non-serpentine systems, $F_r$ con $F_r'$ e la 9d) con la A3).

## Appendix A2

Problem related to the selection of a method and a step for integration to find solution for equation (1) yielding accurate results will be dealt in this section. Since in our case the parameter of interest is the index I, the equation has been integrated by gradually reducing the progression of that index as a function of the integration step. Amongst various numerical methods available, the second Runge Kutta method (RK2), the fourth order Runge Kutta method (RK4) was used. For illustration of these methods refer to (Pasquini, 1976). Calculation were made for flow in the "on - off" operation for 0.014 kg/sec m$^2$, common for conventional thermal solar systems. For convenience sake, formulas that defines the index I, is again reported, as given below.

$$I = 100(Ropt - Rc)/Rc$$

The following (table 2) shows *Ropt Ropt* and $R_c$ on an annual basis and index I calculated for different values of the integration step using the RK2 method.

**Table 2. *Ropt Ropt* and $R_c$ on an annual basis, and index I**

| Integration Steps (sec) | *Ropt* (euro) Annual | *Rc* (euro) Annual | I (%) Annual |
|---|---|---|---|
| 3600 | 3690 | 3456 | 6.77 |
| 450 | 3435 | 3193 | 7.58 |
| 225 | 3414 | 3172 | 7.63 |
| 112 | 3403 | 3160 | 7.69 |

From the examination of the data contained in the table, we decided to use the RK2 method with integration step of 112 sec for guaranteed approximation of I to the first decimal place, sufficient for our purposes. Finally, to verify and test the goodness of choice, Ropt was calculated by solving (1) using Runge Kutta classic method (RK4), a fourth order algorithm and more accurate than the method RK2. The following (table 3) compares the two methods used (RK4 and RK2).

**Table 3. (RK4 and RK2) Methods**

| Integration Steps (sec) | *Rop* (Euro) Calculation done with RK2 | *Rop* (Euro) Calculation done with RK4 |
|---|---|---|
| 3600 | 3690 | 3703 |
| 450 | 3435 | 3434 |
| 225 | 3414 | 3411 |
| 112 | 3403 | 3400 |

It is evident from the results presented that RK2 and RK4 are practically coincident except for very high integration steps.

In conclusion, it is possible to say that non-high order methods (first, second order) and integration steps of around 2 minutes provide a fairly accurate result for our purposes. This is due to the fact that the high thermal inertia of the boiler results in slow variations in the amplitude of the reservoir temperature so that it can be evaluated with good approximation also by using considerably approximate formulas and times of integration of the order of the minutes.

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